Data Fusion BAUE Estimation of a deterministic vector, applications to image noise and blur reduction

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Abstract. In this work[1] we conceive *centralized data fusion* as a deterministic parameter estimation problem. Two different criterions are compared: best affine unbiased fusion rule (BAUE), and Maximum Likelihood for Gaussian measurement noise. Estimates are described in terms of their covariance matrices, the Cramer-Rao lower bound and simulations. The developed fusion rules are suited to two different image fusion cases: noise reduction under differently exposed images, and blur reduction based on lens response knowledge.

Keywords: multi-sensor fusion, image fusion, BAUE, maximum likelihood, MMSE estimation.

1 Introduction

Data fusion techniques deal with the problem of combining information obtained from several sensing devices, with the purpose of making an optimal estimation of some process characteristic by using all available information. This estimation should be more precise than the observation available in each individual device.

In the development of any data fusion scheme, the first step is to define how the individual measurements are modelled. In the literature, the most adopted observation model gives a linear relationship between the observations and the estimated variables

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \eta, \qquad (1)$$

where $\mathbf{x} \in \Re^N$ is the process characteristic to be estimated, $\mathbf{A} \in \Re^{M \times N}$ represents the observation system, and $\eta \in \Re^M$ is an observation aditive noise. One question that should arise is whether our model considers $\mathbf{x}$ a deterministic or a random variable. The most common assumption is that $\mathbf{x}$ is a random variable, and the problem of data fusion is to find an optimal estimation, or *fusion rule*,

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for $\mathbf{x}$ given the measurements $\mathbf{y}$. However, this work deals with an alternate approach, in which $\mathbf{x}$ is considered to be deterministic and only the noise vector η is random. The fusion is thus stated in terms of a deterministic parameter estimation problem.

In order to recover information of all the components of vector $\mathbf{x}$, the observation matrix $\mathbf{A}$ should represent a *good observation* of the characteristic to be estimated. This means that among all M components of $\mathbf{y}$ there must be at least N linearly independent equations involving the components of $\mathbf{x}$, in other words, $Rg(\mathbf{A}) = N$. However, if $M = N$, there's no redundancy in the measurements, no data fusion occurs and the solution is trivial. Consequently, our attention will be put in the problem with $M > N$ and $Rg(\mathbf{A}) = N$.

The next step is to choose an optimization criterion. The most widely adopted criterion is minimum mean square error (MMSE), and during the last two decades, extensive research has been done on fusion according to this criterion, specially on the multiple sensor target tracking [1], [2], Decentralized Kalman Filters fusion [3], [4], [5], etc. These approaches consider $\mathbf{x}$ to be a random process, and its time dependency is modelled by a linear system equation. Apart from MMSE, other criterions were proposed, such as minimum entropy [6] and maximum likelihoood [7], [8]. This paper deals with two different criterions, both under the assumption that $\mathbf{x}$ is deterministic. In section 2, an unbiased affine MMSE fusion rule will be developed. In section 3, a Maximum Likelihood fusion rule will be obtained for the case of Gaussian noise. Descriptions of the results will be given, in terms of covariance matrices and the Cramer-Rao lower bound, and performance curves will be presented in section 4.

Under these hypothesis, two image fusion schemes will be presented, in section 4, by which several pictures with different exposure and noise conditions will lead to a noise reduced fused image, and also a blur reduction method will be presented. In section 5, conclusions of this work will be sumarized.

2 MMSE fusion rule

Under the hypothesis that $\mathbf{x}$ is deterministic, the minimum mean squared error fusion rule, $\mathbf{x}_F$, is obtained as a result of minimizing the following measure:

$$MSE = E\left[\|\mathbf{x}_F - \mathbf{x}\|^2\right] \tag{2}$$

In this section, a Mahalanobis norm will be adopted, by using some positive-definite symmetrical matrix $\mathbf{M}$:

$$\|\mathbf{z}\|_m^2 = \mathbf{z}^T\mathbf{M}\mathbf{z}$$

This norm defines which components of $\mathbf{z}$ are more heavily weighted, and thus adds generality to our optimization results. The problem will be stated for the case where $\mathbf{x}_F$ is an unbiased estimate of $\mathbf{x}$, and our study will be limited to affine fusion rules.

2.1 Unbiased affine rule (BAUE)

Considering an observation model such as (1), where noise moments of first and second order are known,

$$E[\eta] = \mathbf{0} \quad and \quad E\left[\eta\eta^T\right] = \Sigma_\eta$$

the aim is to develop an unbiased affine fusion rule of the form

$$\mathbf{x}_F = \mathbf{B}\mathbf{y} + \mathbf{u} \tag{3}$$

with $\mathbf{B} \in \Re^{N\times M}$ and $\mathbf{u} \in \Re^N$, such that the mean squared error is minimum:

$$\hat{\mathbf{x}}_F = \hat{\mathbf{B}}\mathbf{y} + \hat{\mathbf{u}} = \arg \min_{\mathbf{x}_F = \mathbf{B}\mathbf{y}+\mathbf{u}} E\left[\|\mathbf{x}_F - \mathbf{x}\|_m^2\right]$$

In order to achieve an unbiased fusion rule, the following condition must be imposed:

$$E[\mathbf{x}_F] = \mathbf{x},$$

and substituting equations (1) and (3) we obtain

$$\mathbf{B}\mathbf{A}\mathbf{x} + \mathbf{B}E[\eta] + \mathbf{u} = \mathbf{B}\mathbf{A}\mathbf{x} + \mathbf{u} = \mathbf{x}.$$

Since neither $\mathbf{B}$ nor $\mathbf{u}$ depend upon $\mathbf{x}$ (i.e. it's an affine rule) it is necessary that:

$$\mathbf{B}\mathbf{A} = \mathbf{I} \quad and \quad \mathbf{u} = \mathbf{0} \tag{4}$$

In order to find the optimum value for $\mathbf{B}$, the mean squared error norm is written in terms of this matrix, and both conditions from (4) are used:

$$\begin{aligned}
E\left[\|\mathbf{x}_F - \mathbf{x}\|_m^2\right] &= E\left[(\mathbf{x}_F - \mathbf{x})^T \mathbf{M}(\mathbf{x}_F - \mathbf{x})\right] \\
&= E\left[(\mathbf{B}\mathbf{y} + \mathbf{u} - \mathbf{x})^T \mathbf{M}(\mathbf{B}\mathbf{y} + \mathbf{u} - \mathbf{x})\right] \\
&= E\left[((\mathbf{B}\mathbf{A} - \mathbf{I})\mathbf{x} + \mathbf{B}\eta)^T \mathbf{M}((\mathbf{B}\mathbf{A} - \mathbf{I})\mathbf{x} + \mathbf{B}\eta)\right] \\
&= E\left[(\mathbf{B}\eta)^T \mathbf{M}(\mathbf{B}\eta)\right] = E\left[Tr\left(\mathbf{M}\mathbf{B}\eta\eta^T\mathbf{B}^T\right)\right] \\
&= Tr\left[\mathbf{M}\mathbf{B}\Sigma_\eta\mathbf{B}^T\right] = Tr\left[\Sigma_\eta^{\frac{1}{2}}\mathbf{B}^T\mathbf{M}\mathbf{B}\Sigma_\eta^{\frac{1}{2}}\right]
\end{aligned} \tag{5}$$

In the above expressions, $Tr(\cdot)$ represents the matrix trace operation. Its cyclic permutation property ($Tr(\mathbf{CDE}) = Tr(\mathbf{DEC}) = Tr(\mathbf{ECD})$) is used, and also its linearity allows interchanging the order between $Tr(\cdot)$ and $E[\cdot]$. Now, a cost function $J(\cdot)$ can be defined, which includes both the mean squared error and the $N \times N$ equations represented by $\mathbf{BA} = \mathbf{I}$, by using a Lagrange multiplier matrix Λ:

$$J(\mathbf{B}, \Lambda) = Tr[\Sigma_\eta^{\frac{1}{2}} \mathbf{B}^T \mathbf{M} \mathbf{B} \Sigma_\eta^{\frac{1}{2}}] + Tr[\Lambda(\mathbf{BA} - \mathbf{I})] \tag{6}$$

The minimum of this cost function can be found diferentiating with respect to $\mathbf{B}$:

$$\frac{\partial J}{\partial \mathbf{B}} = 2\mathbf{M}\hat{\mathbf{B}}\Sigma_\eta + (\mathbf{A}\Lambda)^T = \mathbf{0}$$

After solving for $\hat{\mathbf{B}}$, and applying the condition associated to the Lagrange multiplier, we obtain

$$\hat{\mathbf{B}} = (\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A})^{-1} \mathbf{A}^T \Sigma_\eta^{-1} \tag{7}$$

and hence, the optimum unbiased affine fusion rule is given by:

$$\hat{\mathbf{x}}_{F_{BAUE}} = \left(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \Sigma_\eta^{-1} \mathbf{y}$$

which is independent on the choice of the norm matrix $\mathbf{M}$, and is the same for any noise distribution. This result is equivalent to the minimum variance result published in [6], where the scalar estimate case, with $N = 1$ and $\mathbf{A}^T = [1, 1, \ldots, 1]$, is solved.

In the development of this fusion rule, it was assumed that the matrix Σ_η^{-1} exists. This is a reasonable assumption, since it represents the fact that the aditive noise in any measurement is statistically different from the noise in every other measurement. If $det(\Sigma_\eta) = 0$, some noise component is a linear combination of the others, which in practice does not occur. It was also assumed that $\left(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A}\right)^{-1} \in \Re^{N \times N}$ exists. This is true since $Rg(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A}) = Rg(\mathbf{A}) = N$, for Σ_η being positive-definite.

In order to measure the performance of our estimate, the covariance matrix of $\hat{\mathbf{x}}_{F_{BAUE}}$ is analysed.

$$\begin{aligned} \Sigma_{F_{BAUE}} &= E\left[(\hat{\mathbf{x}}_{F_{BAUE}} - \mathbf{x})(\hat{\mathbf{x}}_{F_{BAUE}} - \mathbf{x})^T\right] \\ &= E\left[\left(\hat{\mathbf{B}}\mathbf{A}\mathbf{x} + \hat{\mathbf{B}}\eta - \mathbf{x}\right)\left(\hat{\mathbf{B}}\mathbf{A}\mathbf{x} + \hat{\mathbf{B}}\eta - \mathbf{x}\right)^T\right] \\ &= E\left[\left(\hat{\mathbf{B}}\eta\right)\left(\hat{\mathbf{B}}\eta\right)^T\right] = \hat{\mathbf{B}}\Sigma_\eta\hat{\mathbf{B}}^T \end{aligned} \tag{8}$$

Then, by replacing the matrix $\hat{\mathbf{B}}$ in accordance to (7) we obtain:

$$\Sigma_{F_{BAUE}} = \left(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A}\right)^{-1} \tag{9}$$

This covariance matrix is a measure of the dispersion, or perturbation, of the estimate as a consequence of the observation noise.

The results in this section are also equivalent to those in [5], referred as "State fusion based on trace of covariance matrix", where an affine unbiased fusion rule is developed for state vectors, supposing independent observations.

3 ML fusion rule

In this section, the problem of developing a fusion rule is stated in terms of the Maximum Likelihood criterion. This criterion, differently from MMSE, considers that the estimated variable is a deterministic parameter of a particular probability distribution, and seeks for the value of $\mathbf{x}$ that maximizes the probability of occurence of the set of observations $\mathbf{y}$, this is:

$$\hat{\mathbf{x}}_F = \arg\max_{\mathbf{x}} \; p(\mathbf{y}|\mathbf{x})$$

The development of a fusion rule under this criterion, is now subject to the distribution of the noise η. The case of Gaussian noise will be analized, and no further assumption will be made on the fusion rule.

3.1 Gaussian noise

Considering, again, an observation model such as (1) and under the Gaussian hypothesis,

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \eta \qquad \textit{with} \quad \eta \sim \mathcal{N}(\mathbf{0}, \Sigma_\eta)$$

a fusion rule $\mathbf{x}_F$ (not necessarily unbiased and affine) that is optimum under the ML criterion is desired. Since $\mathbf{x}$ is a deterministic parameter, the observations $\mathbf{y}$ are distributed according to $\mathcal{N}(\mathbf{A}\mathbf{x}, \Sigma_\eta)$.

Firstly, the likelihood function is defined:

$$\begin{aligned} L_{\mathbf{x}}(\mathbf{y}) &\doteq p(\mathbf{y}|\mathbf{x}) = \\ &= \frac{1}{(2\pi)^{M/2}|\Sigma_\eta|^{1/2}} e^{-\frac{1}{2}(\mathbf{y}-\mathbf{A}\mathbf{x})^T \Sigma_\eta^{-1}(\mathbf{y}-\mathbf{A}\mathbf{x})} \end{aligned} \tag{10}$$

which maximum can be found by maximizing the Log-Likelihood

$$l_{\mathbf{x}}(\mathbf{y}) \doteq \ln\left(L_{\mathbf{x}}(\mathbf{y})\right) = k - \frac{1}{2}(\mathbf{y} - \mathbf{A}\mathbf{x})^T \Sigma_\eta^{-1}(\mathbf{y} - \mathbf{A}\mathbf{x})$$

Hence, after imposing $\frac{\partial l_{\mathbf{x}}(\mathbf{y})}{\partial \mathbf{x}} = \mathbf{0}$, the following equation is found

$$-2\mathbf{A}^T \Sigma_\eta^{-1}(\mathbf{y} - \mathbf{A}\hat{\mathbf{x}}_F) = \mathbf{0}$$

which leads to the optimal fusion rule:

$$\hat{\mathbf{x}}_{F_{ML}} = \left(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \Sigma_\eta^{-1} \mathbf{y} \tag{11}$$

This result shows that the Maximum Likelihood fusion rule for the case of Gaussian noise is the same as the unbiased MSE affine fusion rule (section 2.1).

Conclusion 1 The fact that this ML fusion rule is unbiased and also has the minimum MSE among all affine unbiased estimates, leads to the conclusion that it is the Minimum Variance Unbiased Estimate (MVUE) for the Gaussian case. This can be easily proved by noticing that the ML estimate depends upon the sufficient statistic [9]:

$$\mathbf{s}(\mathbf{y}) = \mathbf{A}^T \Sigma_\eta^{-1} \mathbf{y} \tag{12}$$

which ensures that the Cramer-Rao lower bound is attained:

$$\begin{aligned} \Sigma_{F_{ML}} &= \left\{ E\left[\left(\frac{\partial l_{\mathbf{x}}(\mathbf{y})}{\partial \mathbf{x}} \right) \left(\frac{\partial l_{\mathbf{x}}(\mathbf{y})}{\partial \mathbf{x}} \right)^T \right] \right\}^{-1} = \\ &= \left(\mathbf{A}^T \Sigma_\eta^{-1} \mathbf{A} \right)^{-1} \end{aligned} \tag{13}$$

In other words, the affine fusion rule (11) is an efficient estimation under the Gaussian assumption.

4 Image fusion

In the observation model described by equation (1), $\mathbf{x}$ can represent the set of light intensities of some *original* image, and $\mathbf{y}$ may thus represent an observation of such image, for example a filtered or noisy version of $\mathbf{x}$. Two fusion techniques are presented along this section, both of which are modelled and solved according to the previously developed unbiased affine fusion rule.

4.1 Image noise reduction

In order to be $M > N$, such as explained in the introduction, $\mathbf{y}$ must contain more than one complete observation of $\mathbf{x}$, so as to have some redundancy and also represent a good observation scheme. In our case, an amount of k different observations will be stacked in vector $\mathbf{y}$:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_k \end{bmatrix}, \tag{14}$$

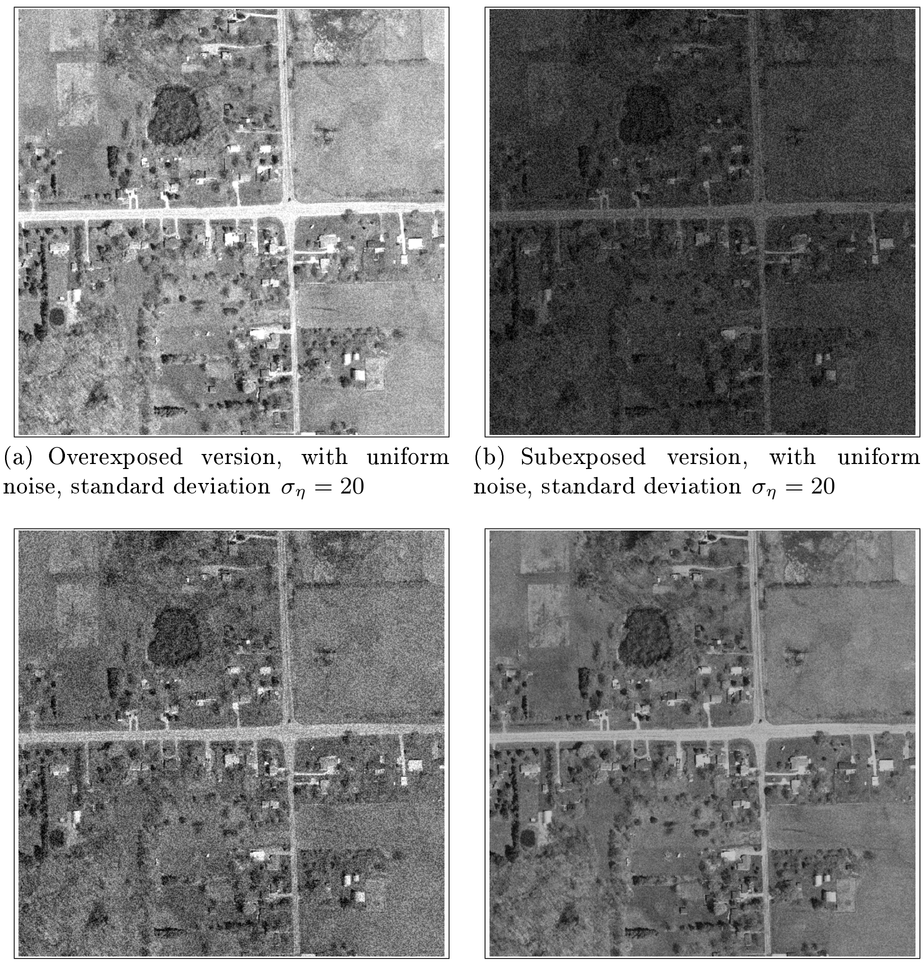
where each observation will be related to the original image by

$$\mathbf{y}_i = \mathbf{A}_i \mathbf{x} + \eta_i, \tag{15}$$

with $\mathbf{A}_i \in \Re^{N \times N}$. These matrices, $\mathbf{A}_i$, contain the models of the the cameras that took each photograph, or any parameter that represents the kind of distortion present on each observation. A widely used linear camera model ("pinhole camera") is described in [10].

In our first simulation, different exposure snapshots are modelled by fixing matrices $\mathbf{A}_i = a_i \mathbf{I}$, being a_i brightness factors. Several uncorrelated noise conditions

($\Sigma_{\eta_i} = E[\eta_i \eta_i^T] = \sigma_{\eta_i}^2 \mathbf{I}$) are modelled, by fixing different standard deviation levels, and by including some images corrupted by Gaussian noise and others by uniform noise. Figure 1 shows three different observed images: 1(a) is an overexposed version, 1(b) is a subexposed one, and 1(c) has normal light exposure. In all cases, uniformly distributed noise is present, with $\sigma_{\eta_1} = 20$, $\sigma_{\eta_2} = 20$ and $\sigma_{\eta_3} = 30$, respectively (in a 256 grayscale image). In the fusion scheme, other three images with same exposure and noise deviations, but with Gaussian noise, were included, summing a total of six observed images. The resulting fused image is that of Fig.1(d), where considerable noise reduction (standard deviation $\sigma_F \sim 9$) and detail recovery are observed.



(a) Overexposed version, with uniform noise, standard deviation $\sigma_\eta = 20$

(b) Subexposed version, with uniform noise, standard deviation $\sigma_\eta = 20$

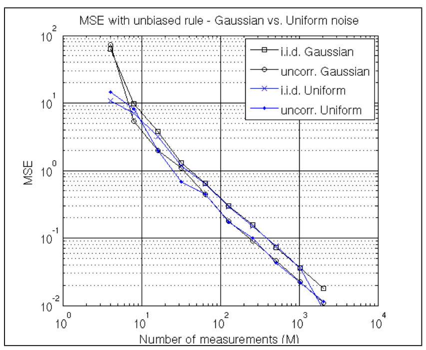
(c) Normal exposure image, with uniform noise, standard deviation $\sigma_\eta = 30$

(d) BAUE fused image

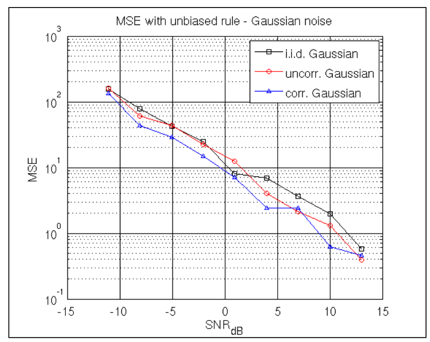
Fig. 1. Observed images and fused image

In order to measure the performance of the above developed fusion rule, different simulations are performed. Parameters, such as number of sensors (M) and observation noise levels (σ_{η_i}) are swept so as to compare how this technique responds.

Number of sensors It is expected that by increasing the number of sensors M (higher redundancy), for a fixed vector $\mathbf{x} \in \Re^N$ and noise condition, an improvement in precision will be gained. For instance, let $N = 4$ and $\mathbf{x} = \frac{1}{2}[1, 1, 1, 1]^T$, and let M change. A first scenario would be that in which noise is independently distributed as $\eta_i \sim \mathcal{N}(0, \sigma_\eta^2)$, and hence $\Sigma_\eta = \sigma_\eta^2 \mathbf{I}$. Under such assumption, the minimum variance unbiased fusion rule is (11). Accordingly, in Fig.2(a) ("i.i.d. Gaussian") the MSE is plotted as a function of the number of sensors M.



(a) MSE vs. M, Gaussian and Uniform noise



(b) MSE vs. SNR, Gaussian noise

Fig. 2. Performance analysis

Another case, in which noises are still uncorrelated and Gaussian but with different variances, $\Sigma_\eta = diag(\sigma_{\eta_1}^2, \sigma_{\eta_2}^2, \ldots, \sigma_{\eta_M}^2)$, is also plotted in Fig.2(a) ("uncorr. Gaussian").

It was proved that the affine rule (11) is the MVUE in the case of Gaussian noise, but for other noise distributions this affine rule (same as in section 2.1) might not lead to the minimum variance estimate. However, since the affine fusion rule developed in section 2.1 only depends on first and second order noise moments, it is expected that for any noise distribution with same mean and covariance matrix, the same performance will be attained. Hence, an equivalent scenario is now simulated, for a uniform noise distribution with same covariance matrices as in the Gaussian case, see Fig.2(a) ("i.i.d. Uniform" and "uncorr. Uniform"). A common characteristic is observed for all the traces:

$$MSE \propto \frac{1}{M} \quad for \quad M > N = 4 \tag{16}$$

and MSE levels under uniform noise are equal to those under Gaussian noise, as was previously mentioned. These characteristics are also observed for different choices of N and $\mathbf{x}$. In all cases, a randomly generated observation matrix $\mathbf{A} \in \Re^{M \times N}$ with $Rg(\mathbf{A}) = N$ was used. Cases with arbitrary noise covariance matrix were not simulated due to numerical instability while inverting Σ_η for high values of M.

Signal to noise ratio It is also expected that the MSE varies depending on the signal to noise ratio (SNR) of the measurements. So, for fixed $M = 12$, and same N, $\mathbf{x}$ and $\mathbf{A}$ as in section 4.1, different noise conditions are simulated. Our definition of SNR is:

$$SNR = \frac{\|\mathbf{x}\|^2}{E[\|\eta\|^2]} = \frac{\|\mathbf{x}\|^2}{Tr\left[\Sigma_\eta\right]} \tag{17}$$

and

$$SNR_{dB} = 10 \cdot \log_{10}(SNR) \tag{18}$$

An example involving Gaussian noise is presented, considering three different noise correlation conditions, Fig.2(b). Such as in the previous section, two traces are shown in correspondence with $\Sigma_\eta = \sigma_\eta^2 \mathbf{I}$ ("i.i.d. Gaussian") and $\Sigma_\eta = diag(\sigma_{\eta_1}^2, \sigma_{\eta_2}^2, \ldots, \sigma_{\eta_M}^2)$ ("uncorr. Gaussian"), and a third one corresponding to an arbitrary noise covariance matrix. The same tendency is observed for all three cases:

$$MSE \propto \frac{1}{SNR} \tag{19}$$

and, again, same MSE levels are attained, independently of the correlation condition. Equivalent results have been obtained under uniformly distributed noise, and other fixed values for M, N and $\mathbf{x}$.

4.2 Image blur reduction

Under the same observation model as in equation (1), a blurring process can be described. The fact that optical lenses have a low pass characteristic, given by their Modulation Transfer Function (MTF, [11]), means that the light intensity of each observed pixel in $\mathbf{y}$ is influenced by the light in the surrounding pixels, and such phenomenon is perceived as blur. Such influence can be considered to be linear and shift invariant, in a first approach:

$$y_{i,j} = \sum_{k,l \in W} a_{k,l} x_{i-k,j-l} \tag{20}$$

where W represents a window with the size of the impulsive response of the lens, $a_{k,l}$. If an amount of N_W coefficients are taken into account, then, by taking at least N_W independent observations of the same pixel, $y_1 \ldots y_{N_W}$, the original light intensity $x_{i,j}$ can be estimated using the BAUE fusion rule. Under this approach, the observation model of equation (1) is applied to each pixel in the image, where $\mathbf{x}$ represents the light intensity of the pixel being fused and those of the surrounding ones, the observation vector $\mathbf{y}$ includes $M \geq N_W$ independent observations of the same pixel, and the matrix $\mathbf{A}$ contains the coefficients of the impulsive response of the lens in each observation. The fact that we need at least N_W independent observations means that the available photographs must be taken with different lens characteristics: by using different lenses, or by using the same lens with different relative apertures.

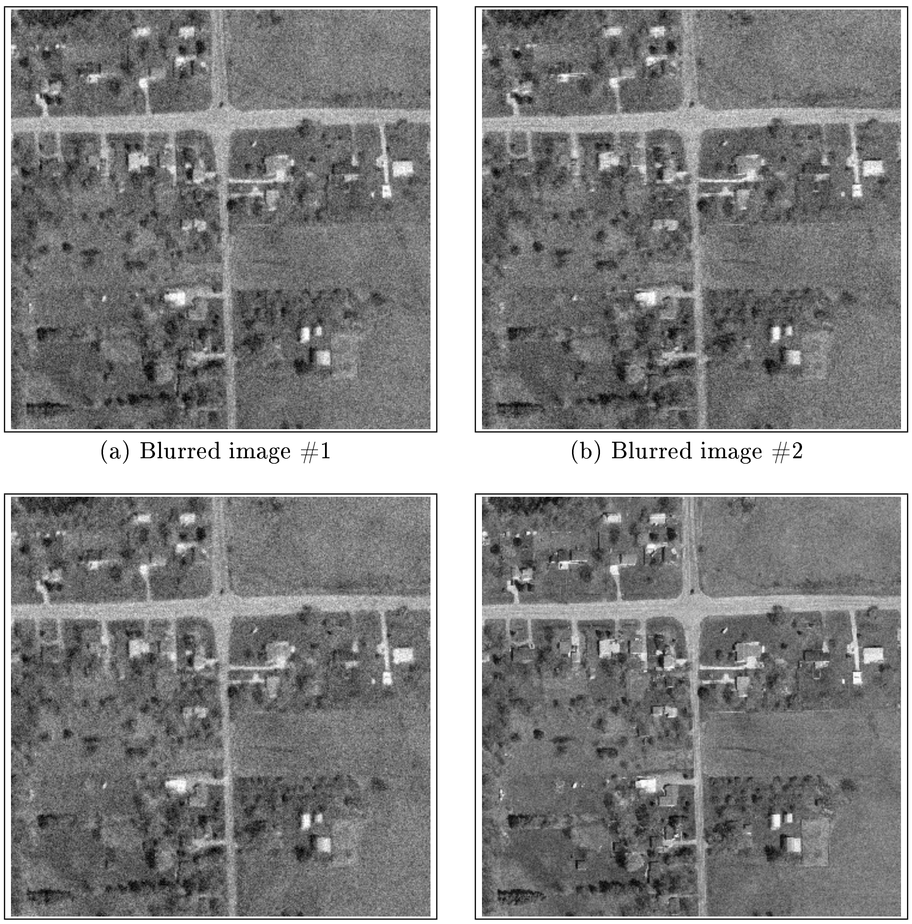
Simulations have been made for $N_W = 5$, which means that the most significant pixels involved in the blurring process are those adyacent to $x_{i,j}$ (above, below, right and left), and different blurring levels were set by fixing the rows of matrix $\mathbf{A}$. Considering a noiseless scenario, when $M = N_W$ the BAUE estimate is nothing but the solution to a linear compatible system, and when $M > N_W$ it is the solution via a least squares approximation. Under such condition, the original image can be recovered completely. Now, when the observations are perturbed by additive noise the BAUE estimate is consequently perturbed, according to equation (9), however the blur reduction is also completely achieved. For each of the lens responses $a_{k,l}$, eight noisy photographs were included in the fusion scheme, with independent and identically distributed Gaussian additive noise, with $\sigma_\eta = 20$. Figures 3(a), 3(b) and 3(c) show three blurred and noisy images, and 3(d) shows the BAUE fused image using all available images. The standard deviation of the error in the fused image is $\sigma_{F_{BAUE}} \sim 12.7$, and further simulations show the same performance tendency as in Fig.2(a).

5 Conclusions

In this paper, an affine unbiased fusion rule has been developed, according to the minimum mean square error criterion. Same results as those for the BAUE fusion rule have been obtained by adopting the Maximum Likelihood criterion for the Gaussian noise case, and the efficiency of the estimate was proved.

The unbiased affine rule was suited to two different image fusion cases: one in which differently exposed and noisy images lead to a noise reduced estimation, and another one in which knowledge of the lens response is used in a blur reduction method. Simulations results show how the performance of the developed fusion rules depends upon the number of sensors and the signal to noise ratio.

Further research is being done on the modelling of the blurring response, $a_{k,l}$, based on MTF curves of lenses, which in practice are nonlinear and highly shift variant.



(a) Blurred image #1 (b) Blurred image #2

(c) Blurred image #3 (d) Recovered image

Fig. 3. Observed images and fused image

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